New Concepts on Caustics and Applications

G.A. Papadopoulos

Department of Engineering Science, Section of Mechanics, National Technical University of Athens, GR-157 73, Zografou campus, Athens, Greece.

E-mails: gpad@central.ntua.gr

Abstract

A new formula of experimental evaluation of the stress intensity factor was proposed depending on the area of the caustic. From series of experiments, which were performed in single edge-cracked specimens made of Lexan (PCBA), it was found that the values of the stress intensity factor $K_{I,}$, which were based on the diameters of the caustics, were varied with the load, the crack length and the caustic shape because the caustic become oval in higher loads or higher crack lengths. This means that the stress intensity values which are evaluated by the two diameters of the caustics (D_t , D_l), are significantly differed. The accurate stress intensity factor K_I can be experimentally evaluated by the area of the formed caustic. According to the new formula of the stress intensity factor of the stress optical constants were evaluated. The present work illuminates the complicated problem of the stress intensity factors evaluation by the caustic diameters and establishes the new accurate of the caustic area approach for the evaluation of the stress intensity factors at crack tip and the stress optical constants of the transparent materials.

Keywords

Cracks; Caustics; Caustics Crea; Stress Intensity Factors; Contact Problems

Introduction

The optical method of caustics, introduced by Manogg (1964) and Theocaris (1970), has been shown to be very effective for the determination of the characteristic parameters of singular elastic fields, where the corresponding stress parameters in these particular regions are governed by singularities which render the solution of the difficult problem, if not impossible, by conventional stress analysis methods. The difficulty arises mainly from the fact that the highly strained region at the singularity is very small and therefore the information gathered by classical experimental methods is rather vague and inaccurate.

According to the method of reflected caustics, a light beam impinges in the immediate vicinity of the singularity and is reflected from the front and rear faces of the plate (for the case of transparent materials). The reflected rays are collected along a singular surface, which is strongly illuminated. A reference screen, placed some distance from the plate, intersects this surface and yields a singular curve, the caustic, which is, for all cases studied up to now, a generalized epicycloid. In this way, and by applying simple laws of geometric optics, that is the reflection laws, the singular stress field is transformed into an optical singularity, the caustic, which yields all the necessary information for evaluation of the stress singularity.

The method of reflected caustics was used for the solution of several elasticity problems of particular interest in engineering applications. The method was applied primarily to cases with singularities, such as those appearing in cracked plates under plane stress conditions; to contact problems; and to problems of multiwedges (composite materials). However, the method works equally satisfactorily in elastic problems with any type of stress concentration, not necessarily including singularities. In such cases the caustic is generated from a deformed boundary instead of from a singular curve in the interior of the plate (initial curve). For a review of all these applications the reader is referred to a review paper by Theocaris (1974, 1977, 1978), Rosakis (1983), Kalthoff (1987), Papadopoulos (1993, 2005, 2011).

The method of caustics was theoretically developed for elastic homogeneous and isotropic materials. For virtual materials, the caustic shape dependes on the material behavior at crack tip at the board of which is defined by the initial curve. Also, the caustic shape dependes on the plastic zone, core region, which is developed at crack tip because of Poisson's phenomenon. The shape of the caustic can be changed by the specimen surface curvature (Theocaris 1976), (Tomlinson and Patterson 1999).

The experimental method of caustics was applied to evaluate the stress intensity factors by the caustic diameters. The values of the stress intensity factors, which were calculated by the diameters of the caustic, (D_t transverse and D_l longitudinal diameter), which

its ratio is $D_t / D_l = 1.056$, are equal, $K_I(D_l) = K_I(D_t)$. While, in higher loads the ratio of diameters is $D_t / D_l < 1.056$, which means $D_t < D_l$, the calculated values of the stress intensity factors are unequal, $K_I(D_l) > K_I(D_t)$. Then, a new formula for the evaluation of the stress intensity factor which was based on the caustic area, was applied, Papadopoulos (2011). By this formula, a new approach to evaluate the stress optical constants, Badalouka et.al (2011). Recent work, Unger et. al. (2005) and Hedans et. al. (2008), has demonstrated that the method of caustics can be used to study the plasticity around the crack tip.

The caustics dependes on the stress optical constants. The stress optical constants of PCBA (Lexan) and PMMA (Plexiglas) were calculated by the strain gauge and interferometric techniques, Raftopoulos et. al. (1976). In a series of recent publication, Konsta-Gdoutos (1996 a.b. 1999) was reported resulting from the character and the extent of a three-dimensional region surrounding the crack tip by the method of caustics. According to these publications the stress intensity factors can be determined even for the case when the initial curve of the caustic lies in the region where the state of stress in three-dimensional cannot be approximated as plane stress or plane strain. Also, in these publications nomograms of stress optical constants versus an empirical factor of triaxiality were given. In this work the stress optical constants were experimentally determined by the reflected caustics. The triaxiality factor can be experimentally determined by the stress optical constants variation according to nomograms of Figs 7 and 8 of Ref. Konsta-Gdoutos (1996b).

Method of Reflected Caustics

The optical method of caustics is able to transform the stress singularity into an optical singularity, using the reflection laws of geometric. For divergent light beam the reflected light rays from front, (f) and rear, (r), face of the plate form two caustics, the caustic (f) and the caustic (r). For a cracked isotropic elastic specimen, the parametric equations of the two caustics are Papadopoulos (1993):

$$X_{r,f} = \lambda_m r_o (\cos\theta \pm \frac{2}{3}\cos\frac{3\theta}{2}) \tag{1}$$

$$Y_{r,f} = \lambda_m r_o (\sin\theta \pm \frac{2}{3}\sin\frac{3\theta}{2}) \tag{2}$$

where r_o is the radius of initial curve of the caustics. This radius is given by:

$$r_o = \left(\frac{3}{2}C_{r,f}\right)^{2/5} \quad with \quad C_{r,f} = \frac{\varepsilon z_o d c_{r,f} K_I}{\lambda_m \sqrt{2\pi}} \tag{3}$$

with:

$$\lambda_m = \frac{z_o \pm z_i}{z_i} \tag{4}$$

where z_o is the distance between specimen and reference plane, d is the thickness of the specimen, λ_m is the magnification ratio of the experimental set-up, z_i is the distance between specimen and light beam focus, $\varepsilon = 2$ for caustic (r) and $\varepsilon = 1$ for caustic (f) and K_I is the stress intensity factor for the mode-I stress state. The c_r and c_f are the stress-optical constants of the material. The stress-optical constant c_f is given by:

$$c_f = \frac{\nu}{E} \tag{5}$$

where ν is the Poisson's ratio and E is the modulus of elasticity of the material.

The experimental stress intensity factor K_I is estimated by the relation, Papadopoulos (1993):

$$K_I = \frac{2\sqrt{2\pi}}{3\varepsilon z_o d \lambda_m^{3/2} c_{r,f}}\left(\frac{D_{t,l}}{\delta_{t,l}}\right)^{5/2} = 1.67C\left(\frac{D_{t,l}}{\delta_{t,l}}\right)^{5/2}$$
$$with\ C = \frac{1}{\varepsilon z_o d \lambda_m^{3/2} c_{r,f}} \tag{6}$$

where $D_{t,l}$ are the maximum diameters of the caustics (D_t transverse and D_l longitudinal) and $\delta_{t,l}$ are the correction factors with:

$$\delta_t = 3.1702 \ and \ \delta_l = 3.00 \tag{7}$$

If the caustic is about circle, Fig. 1 (theoretical castic), the diameters ratio is:

$$\frac{D_t}{D_l} = \frac{\delta_t}{\delta_l} = 1.056 \tag{8}$$

Then, the K_I values are: $K_{I(D_t)} = K_{I(D_l)}$. If the caustic becomes oval, Fig.1 (experimental caustic), $\frac{D_t}{D_l} \neq 1.056$, the K_I values are difference , $K_{I(D_t)} \neq K_{I(D_l)}$. In this state, the relation between the values of the K_I is:

$$K_{I(D_t)} = \left(\frac{D_t / D_l}{1.056}\right)^{5/2} K_{I(D_l)} \tag{9}$$

Fig.1 illustrates the theoretical and the experimental caustics at notch tip for notch length $\alpha = 0.069m$ for the stress $\sigma = 2.42\,MPa$.

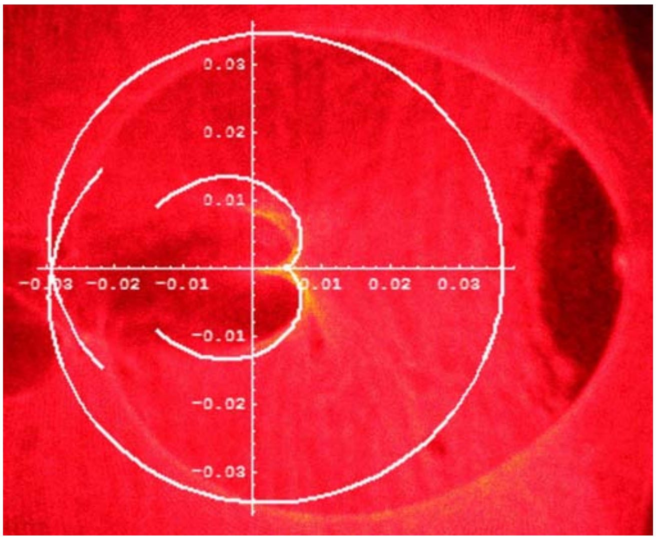

FIG. 1 THEORETICAL AND EXPERIMENTAL CAUSTICS IN LEXAN (BCBA) SPECIMEN WITH STRESS $\sigma = 2.42\,MPa$, FOR NOTCH LENGTH $\alpha = 0.069m$

Because of the shape of caustic is about ellipse, its area is given by the relation:

$$A = \frac{\pi}{4} D_t D_l \tag{10}$$

According to caustic theory, Papadopoulos (1993), the diameters of the caustic are:

$$D_t = 3.1702\lambda_m r_0, \quad D_l = 3.00\lambda_m r_0 \tag{11}$$

Then, from relations (3), (6), (10) and (11), the stress intensity factor K_I becomes:

$$K_I = 0.1358CA^{5/4} \tag{12}$$

Then, the energy release rate or the J integral is given by the relation:

$$G_I = J_I = \frac{K_I^2}{E} = \frac{0.0184C^2A^{5/2}}{E}, \quad \textit{for plane stress state} \tag{13}$$

Fig.2 illustrates the experimental stress intensity factors (normalized to C) calculated by the diameters, $K_{I(D_t)}, K_{I(D_l)}$ and area, A, of the caustic versus the tensile stress for the same crack length $\alpha = 0.040m$. From this figure, it appears that the curve of the stress intensity factor, which was calculated by the area of the caustic, lies between the two others. From these curves a linear variation and values coincidence of the stress intensity factor up to $\sigma = 12MPa$ can be observed. After this stress, a non linear variation and values deviation can be observed. This means that the stress intensity factors values in this stress region, which were calculated with the area of the caustic, are more accurate than the others.

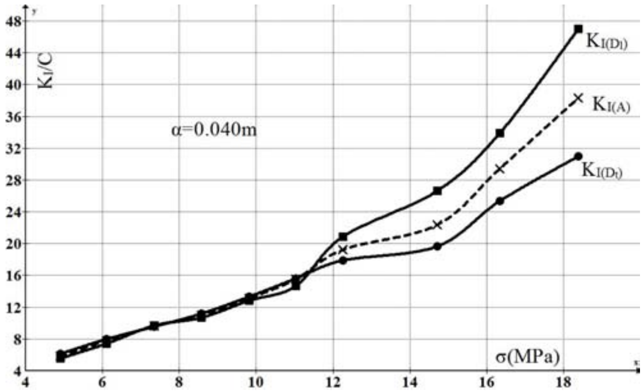

FIG.2 VARIATION OF THE STRESS INTENSITY FACTORS (NORMALIZED TO C) CALCULATED BY THE DIAMETERS, $K_{I(D_t)}, K_{I(D_l)}$ AND AREA, A, OF THE CAUSTIC vs THE TENSILE STRESS FOR THE CRACK LENGTH $\alpha = 0.040m$

Evaluation of the stress optical constants

The c_r and c_f are the stress-optical constants of the material. The stress-optical constant c_f is given by: $c_f = \nu / E$, where ν is the Poisson's ratio and E is the modulus of elasticity of the material. The stress optical constant c_f can be calculated according to method of ref. Theocaris (1979) or by the method of strain gauge. The plotting of the two caustics for divergent light beam is illustrated in Fig. 3.

From equations (1) and (2), according to Fig. 3 for $\theta = 0$, we get:

$$c_r = \frac{c_f}{2\left[5 - 8.18\Delta x / A_{(r)}^{1/2}\right]^{5/2}} \tag{14}$$

where Δx is the distance (BA) and $A_{(r)}$ is the area of the (r) caustic. By the relation (3) the ratio of the stress-optical constants c_r, c_f is estimated. Analogous relation to Rel.(14) from Fig. 4 can be written by combination of caustics (r) and (f) for convergent light beam.

$$c_r = \frac{c_f}{2}\left[5 - 8.18\Delta x / A_{(f)}^{1/2}\right]^{5/2} \tag{15}$$

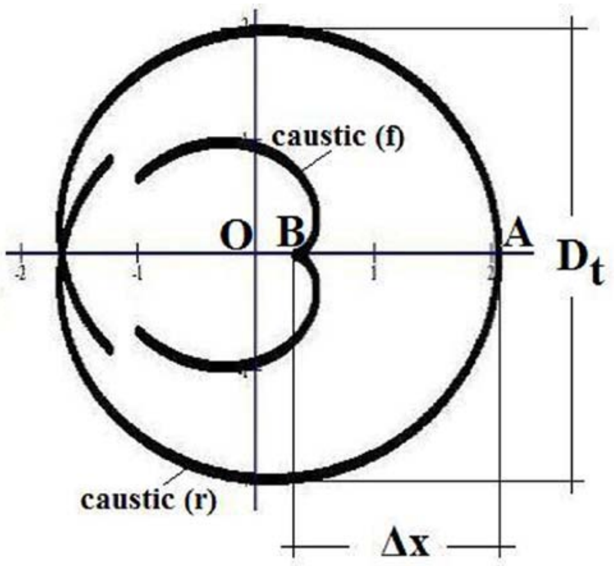

FIG. 3 GEOMETRY OF CAUSTICS (r) AND (f), FOR DIVERGENT LIGHT BEAM

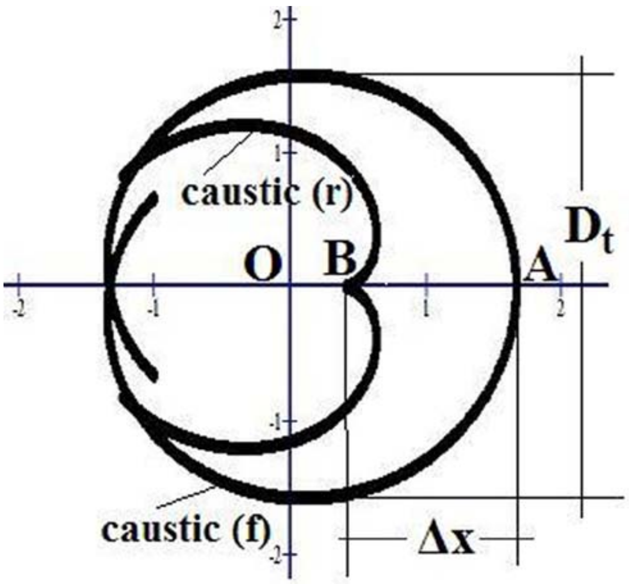

FIG. 4 GEOMETRY OF CAUSTICS (r) AND (f), FOR CONVERGENT LIGHT BEAM

The experimental stress intensity factor K_I is estimated by the relation (6).

From relations (6) and (14), the accurate stress intensity factor can be estimated from Poisson's ratio ν and the modulus of elasticity E or the stress-optical constant c_f and the caustic area:

$$K_I = \frac{0.1358}{z_o d \lambda_m^{3/2} c_f}\left(5A_{(r)}^{1/2} - 8.18\Delta x\right)^{5/2}, \quad c_f = \frac{\nu}{E} \tag{16}$$

Conclusions

The proposed new formula, relation (12), which depends on the area of the caustic, was given more accurate and stable values of the stress intensity factors, as it was concluded from Fig. 2. The stress intensity factor becomes independent from the diameters of the caustic, which were in difficultly calculated if the caustic is not symmetric.

The accurate values of stress optical constants for transparent materials can be experimentally determined by the reflected caustics. The stress optical constants dependes on triaxiality at crack tip region. From the variation of the values of stress optical constants and the nomograms of ref., Konsta-Gdoutos (1996), the triaxiality factor k can be experimentally determined. The stress intensity factors can be accurately determined from accurate values of the stress optical constants.

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